

## SEPARATION LOGIC EXERCISES, LECTURE 4

**Exercise 1.** Give a resource algebra  $\mathcal{M}_t$  with  $t \in \mathcal{M}_t$  s.t.  $\mathcal{V}(t)$  and  $\neg\mathcal{V}(t \cdot t)$ .

**Exercise 2.** Give a resource algebra  $\mathcal{M}_{sf}$  with  $S, F \in \mathcal{M}_{sf}$  such that:

$$\mathcal{V}(S) \qquad \neg\mathcal{V}(S \cdot F) \qquad F = F \cdot F \qquad S \rightsquigarrow F$$

**Exercise 3.** Give a lock invariant using  $\mathcal{M}_{sf}$  to prove the program with  $\mathbf{r} := !\mathbf{r}$  and  $\mathbf{r} := 1$ , then provide a proof sketch of the program.

**Exercise 4.** Give a RA  $(\mathcal{H}_{\{0,1\}}, \cdot, \mathcal{V})$  with  $h_0, h_1 \in \mathcal{H}_{\{0,1\}}$  s.t.

$$\mathcal{V}(h_0 \cdot h_0) \qquad h_0 \cdot h_0 \rightsquigarrow h_1 \cdot h_1 \qquad \forall i, j \in \{0, 1\} \quad \mathcal{V}(h_i \cdot h_j) \Rightarrow i = j$$

**Exercise 5.** Give a lock invariant using  $\text{option } \mathcal{H}_{\{0,1\}} \times \text{option } \mathcal{H}_{\{0,1\}}$  to prove the program with twice  $\text{incr } \mathbf{r}$ , then provide a proof sketch of the program.

**Exercise 6.** In the RA of partial functions  $\mathbb{N} \rightarrow \mathcal{M}_{\text{sf}}$ , show  $\forall \gamma. \neg(\emptyset \rightsquigarrow \{(\gamma, S)\})$ .

**Exercise 7.** Give a definition for  $l \rightsquigarrow \text{Lock } R$  and prove the specs for locks.

**Exercise 8.** Complete the following implications:

$$\forall lvQ \quad \dots \quad -* (\dots) \quad -* \text{wp}(!l) Q$$

$$\forall llv'Q \quad \dots \quad -* (\dots) \quad -* \text{wp}(l := v) Q$$

$$\forall vQ \quad \dots \quad -* (\dots) \quad -* \text{wp}(\text{ref } v) Q$$

**Exercise 9.** Give an invariant using the authoritative resource algebra in order to prove the monotonic counter program. Give specs for each function, and give a proof sketch.